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Fairness and voting

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Abstract In this paper we present a two-period model in which we examine how concern about fairness might affect voter behavior. We show that in the first period politicians choose the median voter's position even if this does not correspond to their bliss points and neither they nor the voters can commit to a particular action. Moreover, concern about fairness creates substantial incumbency advantages. Our results hold even if voters care very little about fairness.

1 Introduction

Numerous experiments have shown that considerations of fairness are sometimes an important factor in determining human behavior.[1] However, there are next to no papers on how the perceived fairness of the policies chosen by politicians affects the behavior of voters.[2] But it does seem plausible that voters do not merely pursue their own material interests when they vote, but also care about fairness. For example, people are sometimes concerned about the

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[1] See for example, the empirical results of Kahneman et al. (1986) on price-setting by firms. Also compare Fehr and Fischbacher (2002) for an overview of the areas where social preferences matter.

[2] An exception is Hahn (2004), who applies the concept of reciprocity introduced by Rabin (1993) and Dufwenberg and Kirchsteiger (2004) to a model of voter behavior. Galasso (2003) and Sausgruber and Tyran (2006) examine the impact of voters' concern for fairness on the level of redistribution policy. There are many papers in the social choice literature that discuss the fairness of allocations or the fairness of procedures (see, e.g., Karni 1996; Chavas and Coggins 2003).

perks and privileges of politicians and may feel dissatisfied when these rents are too high. This might also affect voters' decisions about who to vote for. In general terms, people may dislike politicians being better off than themselves, e.g., because they believe that politicians pursue policies that are in their own interests or that they do not work hard enough. In this paper, we show that considerations of fairness may provide a potential mechanism for the way in which past behavior of politicians may affect voter behavior. It may also induce politicians to adopt the median voter's preferred policy, even if they are not able to commit to a particular policy in advance. Thus our model yields a new variant of the Median Voter Theorem.

It is well-known that political parties may adopt the median voter position if they can commit to future policy in advance. This observation is known as the "Median Voter Theorem" (cf. Downs 1957). If, however, political parties cannot commit to following a particular policy in the future and voters are standard rational utility-maximizing agents, then parties have no incentive to pursue the median voter's preferred policy unless political parties are not interested in the policies implemented. One way of motivating convergence between political parties is to consider reputation-building in an infinite-horizon framework (see Alesina 1988).

If the time horizon is not infinite and voters are standard rational utility-maximizers who cannot commit to a specific voting behavior, then the re-election of an incumbent can only depend on the past behavior of a politician if the politician's behavior has revealed new information about the politician's characteristics (e.g. her competence or preferences). However, misconduct or abuse by politicians represent sunk costs and normally cannot be punished by voters if they cannot commit to a certain voting behavior in advance. Thus elections could not be used to discipline politicians in such a model, although disciplining politicians by holding them accountable is usually thought to be one of the important functions of elections (see e.g. Persson et al. 1997). In the literature, retrospective voting rules are thus frequently adopted (see, among others, Barro 1973; Ferejohn 1986; Persson et al. 1997; Persson and Tabellini 2000). Voters are assumed to be able to commit to a voting rule at the beginning of the period, which enables the election to discipline the policy-maker. However, this may seem somewhat ad hoc and hence unsatisfactory, as it is not clear how voters should commit to a (time-inconsistent) behavior. Accordingly, this paper can be viewed as a microeconomic foundation for retrospective voting, as in our model the politician's decision in period one affects her chances of being re-elected in the next period without resorting to the ad hoc assumption of retrospective voting. Past decisions of politicians make them better off or worse off compared to voters. This may, e.g., induce voters to vote against the incumbent party if they feel jealous of its members because it has adopted a policy that was in its members' own interests. By dismissing the incumbent party, voters may thus prevent it from pursuing its interests for another period.

There are several approaches to integrating fairness and reciprocity into game theory, among them Rabin (1993), Levine (1998) and Bolton and Ockenfels (2000). The concept of Fehr and Schmidt (1999) has the advantage of being technically easy to handle, because standard equilibrium concepts like Nash equilibrium can be applied. They introduce an additional term to people's utility, namely that people care not only about their own payoffs but also about the fairness of outcomes. People are assumed to focus on self-centered inequity, i.e., they dislike others being better off than themselves and may also dislike other people being worse off than themselves (the latter effect usually being less pronounced).

An alternative way of integrating considerations of fairness into models of voting would be to use the model by Rabin (1993) or the generalization to dynamic games by Dufwenberg and Kirchsteiger (2004).[3] This avenue is pursued by Hahn (2004). He finds that politicians move closer to the median voter's position when reciprocal motives towards politicians are important. He also shows that the incumbent has a strong advantage.

Our paper is organized as follows. In Sect. 2 we present a model where politicians may choose from an interval of possible policies. In the following section, we construct the equilibrium without fairness and show that politicians cannot be motivated to choose the median voter's preferred policy. In Sect. 4 we discuss how the concept of fairness presented by Fehr and Schmidt (1999) can be integrated into our model. The solution with reciprocity is proposed in Sect. 5. In Sect. 6 we discuss the robustness of our results and present our conclusions in Sect. 7.

2 Model

We assume that there is a continuum of potential policies in each period, represented by the interval $[-1;+1]$. There is an odd number N of voters who are characterized by their bliss points τ. The ideal positions of voters τ are equally spaced on the interval $[-1;+1]$, i.e., the set of voters is $\mathcal{N} := \left\{-1, -1+\frac{2}{N-1}, \ldots, 0, \ldots 1-\frac{2}{N-1}, 1\right\}$. The utility of voters is $u_\tau^{(t)} = -\left(\tau - p^{(t)}\right)^2$ in each period t.[4] We assume that the discount factors amount to $\delta = 1$. This assumption is not crucial but simplifies our analysis. $p^{(t)}$ denotes the policy pursued by the government in period t. We assume that there are two politicians (or political parties), a left-wing politician, L, and a right-wing politician, R. Politicians are not interested in holding office per se, but have utility functions $u_{\tau_L}^{(t)}$ and $u_{\tau_R}^{(t)}$ with $\tau_L = -P$ and $\tau_R = +P$, respectively ($P \in]0,1]$).[5] Their discount

[3] Another generalization of the basic concept has been introduced by Falk and Fischbacher (2006).

[4] Note that our assumption that the bliss points are equally spaced is not important. Our results hold for arbitrarily distributed ideal positions as long as the median voter's position is not too far away from $\tau = 0$.

[5] The symmetry of the politicians' bliss points simplifies the analysis considerably, otherwise the equilibrium values cannot be determined analytically. However, it seems unlikely that our findings would change qualitatively if politicians' bliss points were somewhat asymmetric. The scenario with $\tau \neq 0$ would be formally equivalent to the case with asymmetric bliss points for politicians.

factors also amount to $\delta = 1$. Hence, politicians are only interested in the type of policies ultimately implemented.[6]

There are two periods. At the beginning of the first period each citizen votes for a politician. The politician who gets the majority of votes is elected president. Then the newly elected president chooses a policy $p^{(1)}$ from the range of possible policies $[-1;+1]$. At the beginning of the second period, voters can either re-elect the president or select the alternative candidate instead. Then the politician again chooses a policy $p^{(2)}$ ($p^{(2)} \in [-1;+1]$). Note that we assume that voters cannot commit to certain behavior in advance.

We will identify subgame-perfect Nash equilibria for the standard case where voters are not affected by fairness and for the case with inequity aversion. As voting games often involve multiple equilibria, we apply the following tie-breaking rule. If a voter is not pivotal, he will always vote for the candidate who will guarantee the largest overall utility when elected. Moreover, we assume that in the first period the median voter ($\tau = 0$) will randomize between both candidates with equal probability if he is indifferent. In the following, we will always refer to this class of equilibria, e.g., when we consider uniqueness.

3 Solution when fairness does not matter

It is easy to show that, if we do not consider fairness, only the following equilibria exist:

Proposition 1 *When voters do not care about fairness, the president will always pursue her preferred policy, i.e. in both periods a left-wing (right-wing) politician chooses $\tau = -P$ ($\tau = +P$). Voters with $\tau < 0$ ($\tau > 0$) will always vote for the left-wing (right-wing) candidate. The median voter with $\tau = 0$ is indifferent between both candidates and votes for each politician with probability $1/2$ in the first period. In the second period he also randomizes between both politicians, voting for the left-wing politician with some probability σ ($\sigma \in [0;1]$) and for the other candidate with probability $1-\sigma$.*

Thus, in the first period, no politician has an incentive to deviate from her preferred policy by adopting a policy that is closer to the median position $\tau = 0$. Since voters are purely forward-looking when casting votes at the beginning of the second period, adopting a more moderate policy will not improve the politician's chances of being re-elected.

4 Introducing fairness into our model

According to Fehr and Schmidt (1999) the utility function of players should be augmented by additional terms describing inequity aversion, i.e., players

[6] For simplicity, we assume that politicians do not vote. This simplifies the analysis but is not essential to our results.

dislike inequity unfavorable to them. In addition, they may dislike or like inequity favorable to them. To keep the analysis as simple as possible, we assume that voters are only interested in inequity between themselves and politicians. This may be justified by the observation that election campaigns and the media often focus heavily on the personalities of politicians. Thus voters may develop stronger feelings towards politicians than towards other voters. We might also incorporate inequity aversion of voters with respect to other voters. However, one might adopt the interpretation that the bliss points τ already incorporate voters' inequity aversion with respect to other voters. Similarly, one might argue that politicians' bliss points $-P$ and P, respectively, already incorporate their concerns about inequity. We follow Fehr and Schmidt (1999) in considering only self-centered inequity aversion, i.e., a player does not care about inequity between two other players.

Fehr and Schmidt (1999) focus on piecewise linear inequity aversion. We consider a very general, non-linear form. We define the overall utility function of voters as

$$\begin{aligned} U_\tau\left(p^{(1)},p^{(2)}\right) &= u_\tau(p^{(1)}) + u_\tau(p^{(2)}) \\ &\quad + f\left(u_\tau(p^{(1)}) + u_\tau(p^{(2)}) - u_{\tau_L}(p^{(1)}) - u_{\tau_L}(p^{(2)})\right) \\ &\quad + f\left(u_\tau(p^{(1)}) + u_\tau(p^{(2)}) - u_{\tau_R}(p^{(1)}) - u_{\tau_R}(p^{(2)})\right) \qquad (1) \end{aligned}$$

The first two terms are the standard intertemporal utility under the assumption that the discount factor is 1. The third term captures aversion with respect to inequity between the utility of politician L and the voter. The fourth term depends on the inequity between the utility of politician R and the voter.

Note that we will make specific assumptions about $f(x)$ later. We implicitly assume that voter's feelings about fairness are equally strong towards both politicians, since we use the same function $f(x)$ for both. This seems plausible in the first period of the game. However, at the beginning of the second period one might argue that feelings should be stronger towards the incumbent, because she is more prominent in the media, whereas the alternative candidate may be relatively unknown. If this effect is not too strong, our results continue to hold.[7]

Note that Fehr and Schmidt (1999) compare payoffs at final nodes to describe inequity. By contrast, we compare the intertemporal utility of agents. This implies that individuals suffering from inequity in one period can be compensated for at a later stage. The following example illustrates why we think this is a plausible approach. One out of two individuals has to perform an unpleasant task in every period, e.g., two students who share an apartment have to clean it every week. Then most people would consider it much more fair for the two individuals to perform the task alternately than for one individual to perform the task in every period. Similarly, it is plausible that someone who has to

[7] In this case, a right-wing (left-wing) politician would choose a moderately leftist (rightist) position in the first period of the game. Incumbents will always be re-elected.

perform the unpleasant task every time, while the other person never does so, would incur additional losses from feeling treated unfairly. However, if both individuals do the task alternately, both individuals may not suffer from instantaneous inequity very much when it is their turn to perform the task, since they know that the other individual also performs the same task regularly.

One might ask whether comparing standard sums of discounted per-period utilities would be appropriate, in particular if we extended our model to more than two periods. Then it might seem reasonable that unequal per-period utility that occurred some periods ago does not affect the behavior of players as strongly as present inequity. One could discount past utility to capture this effect. Inequity, i.e., the argument of $f(\cdot)$, could be measured by comparing expressions of the form $\sum_{t=0}^{\infty} \delta^t u_t + \sum_{t=1}^{\infty} \rho^t u_{-t}$ $(0 < \delta < 1, 0 < \rho < 1)$, where the first term describes present and future per-period utility, whereas the second sum captures past utility. The factor ρ is introduced to take account of the fact that the memory of past inequity may fade away as time passes. However, we refrain from pursuing these considerations as they do not seem as crucial for a two-period model as for a multi-period model.

We now have to make some assumptions about $f(\cdot)$, which measures the disutility (or utility) from unequal outcomes. We make the following assumptions:

$$f(0) = 0 \tag{2}$$

$$f'(x) > 0 \quad \forall x < 0 \tag{3}$$

$$f''(x) < 0 \quad \forall x \tag{4}$$

$$f'(x) \geq -\frac{1}{2} \quad \forall x \geq 0 \tag{5}$$

$$f(-x) < f(x) \quad \forall x > 0 \tag{6}$$

Note that $x > 0$ describes a case of inequity favorable to the individual, whereas $x < 0$ captures a case of unfavorable inequity. The first assumption is a normalization and implies that the additional utility or disutility from inequity aversion is zero if there is no inequity. The second condition holds because decreasing unfavorable inequity lowers the losses from inequity aversion. The third assumption implies increasing marginal losses from unfavorable inequity. It also implies decreasing marginal gains from profitable inequity or increasing marginal losses from profitable inequity, respectively. The fourth assumption guarantees that a player cannot increase his overall utility by “throwing away” some of his utility when there is inequity that is favorable to him. Note that the overall change in utility amounts to $-\Delta u - 2f'(x)\Delta u$ when a marginal unit Δu of per-period utility is thrown away and favorable inequity x is identical towards both other players. If our assumption holds, this expression is weakly negative. The last assumption describes the fact that for any amount of inequity x the player will prefer a situation where the inequity is to his own advantage to the case where the same amount of inequity is to his disadvantage.

Additionally, we assume that either $f(x) > 0 \; \forall x > 0$ or $f(x) < 0 \; \forall x > 0$. Thus we distinguish between the case where advantageous inequity is deemed

Fig. 1 An example of the function $f(x)$ for voters who like being better off than others

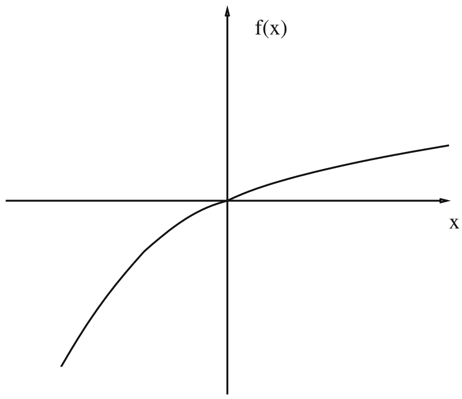
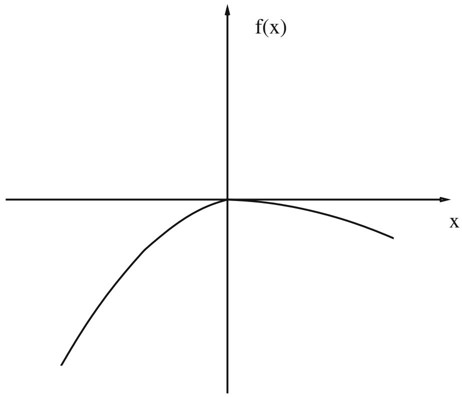


Fig. 2 An example of the function $f(x)$ for voters who dislike being better off than others



desirable and the case where people feel bad if someone else is worse off than themselves.

In Figs. 1 and 2 we demonstrate two examples of $f(x)$. In the first figure, voters like being better off than others, while in the second example voters incur losses if others have lower payoffs than themselves. Note that in both cases voters dislike others being better off.

5 Solution when fairness matters

Now we solve the model for inequity-averse voters. Note that any politician will always choose her preferred policy in the second period of the game, i.e., a left-wing politician will choose $-P$, whereas a right-wing politician will choose $+P$.

Let us assume, without loss of generality, that the right-wing politician has been elected in the first period. Now we consider the decision of voter τ at the beginning of the second period. We assume that, even if he is not pivotal, each voter will always vote for the candidate who will guarantee the largest overall utility when elected. If the incumbent is re-elected, voter τ's utility in the second period is given by

$$\begin{aligned}U_\tau(p,+P) &= -(P-\tau)^2 + f\left(-(p-\tau)^2 - (P-\tau)^2 + (p-P)^2 + 0^2\right)\\ &\quad + f\left(-(p-\tau)^2 - (P-\tau)^2 + (p+P)^2 + 4P^2\right)\\ &= -(P-\tau)^2 + f\left(2(\tau p - \tau^2 + \tau P - Pp)\right)\\ &\quad + f\left(2(\tau p - \tau^2 + \tau P + Pp + 2P^2)\right)\end{aligned}$$

where p is the policy implemented by the right-wing politician in the first period of the game. Note that we have applied the fact that the right-wing politician will choose $+P$ in the second period of the game.

When the alternative candidate is elected at the beginning of the second period, voter τ's utility amounts to

$$\begin{aligned}U_\tau(p,-P) &= -(P+\tau)^2 + f\left(-(p-\tau)^2 - (P+\tau)^2 + (p-P)^2 + 4P^2\right)\\ &\quad + f\left(-(p-\tau)^2 - (P+\tau)^2 + (p+P)^2\right)\\ &= -(P+\tau)^2 + f\left(2(\tau p - \tau^2 - \tau P - Pp + 2P^2)\right)\\ &\quad + f\left(2(\tau p - \tau^2 - \tau P + Pp)\right)\end{aligned}$$

Thus voting for the incumbent is desirable if the following difference in losses is positive:

$$\Delta U(p,\tau) := U_\tau(p,+P) - U_\tau(p,-P) \tag{7}$$

Note that $\Delta U(p,\tau) = -\Delta U(-p,-\tau)$ due to the symmetry of the problem.

The following proposition will prove useful:

Proposition 2 *If the right-wing (left-wing) candidate has been elected in the first period, no voter with* $\tau \le 0$ $(\tau \ge 0)$ *will vote for her if she chooses a policy* $p > 0$ $(p < 0)$ *in the first period.*

The proof is given in the appendix. It follows immediately that a politician will not be re-elected if she chooses her favorite policy $-P$ or $+P$, respectively.

Intuitively, if a politician chooses a policy that is advantageous to herself, then she will make the voter with $\tau = 0$ jealous. If the politician were re-elected, she would implement a policy that would again be beneficial to herself. This, in turn, would make the median voter even more jealous. Thus the voter with $\tau = 0$ would prefer the other candidate to be elected, as she has not benefitted from the policy in the first period. By dismissing a politician who has adopted a very selfish policy, the voter with $\tau = 0$ can achieve an intertemporal balancing of inequity, as it seems more fair to the voter that each candidate should enjoy favorable policy in one period and suffer from detrimental policy in the other period, as opposed to the case where one politician enjoys an advantageous policy in both periods while the other candidate always suffers from policy that she dislikes.

In the Appendix we prove the following proposition:

Proposition 3 *If a right-wing (left-wing) incumbent chooses* $p = 0$ *in the first period, then all voters with* $\tau > 0$ ($\tau < 0$) *will vote for her. The voter with* $\tau = 0$ *will always be indifferent between both politicians if the office holder in the first period chooses* $p = 0$.

We now argue that it is optimal for a right-wing candidate to choose $p = 0$ in the first period if this ensures re-election, instead of choosing P and not being re-elected. The gain in utility in the first period that is the result of choosing P instead of $p = 0$ amounts to P^2. The loss in utility as a consequence of the policy pursued by the left-wing politician in the second period is given by $(2P)^2$. Hence any politician would choose $p = 0$ if this enabled her to be re-elected.[8]

In the Appendix we also prove the following proposition:

Proposition 4 *If a right-wing (left-wing) incumbent chooses a slightly negative (positive)* p *in the first period, then all voters with* $\tau \geq 0$ ($\tau \leq 0$) *will vote for her in the second period.*

Since, for $p = 0$, the median voter is indifferent with respect to both candidates, one might think that equilibria might exist, where the incumbent is not re-elected. This, however, is not the case. A right-wing candidate can always choose a p slightly smaller than 0. Then ΔU is strictly positive for the median voter and all voters with $\tau > 0$. Consequently, the right-wing candidate will always be re-elected. A similar argument holds for the left-wing politician. This is an important result, as it illustrates the advantage of the incumbent politician, who is always re-elected.

In the first period of the game, it is obvious that all voters with $\tau < 0$ will vote for the left-wing politician whereas all voters with $\tau > 0$ will vote for the right-wing politician. In the first period, the median voter is indifferent with regard to both candidates. According to our assumption, he will randomize between both candidates with equal probability.

We obtain the following proposition:

Proposition 5 *For the game with inequity aversion, the following unique equilibrium exists: at the beginning of the first period all voters with* $\tau < 0$ ($\tau > 0$) *vote for the left-wing (right-wing) candidate. The median voter with* $\tau = 0$ *randomizes between both candidates with equal probability. At the beginning of the second period, a left-wing (right-wing) incumbent is re-elected iff she has chosen a policy* $p \geq 0$ ($p \leq 0$). *Both types of politicians choose* $p = 0$ *in the first period*

[8] If the discount factor were very small, no politician would choose $p = 0$ to ensure re-election. However, note that by incurring losses in the first period the politician can prevent losses in the second period from being four times as high.

and are thus re-elected. In the second period, politicians always pursue their own interests, i.e., they opt for $-P$ or $+P$, respectively.

The proposition can be thought of as a variant of the Median Voter Theorem. In the first period, the median voter's position is adopted no matter which politician is elected.

If a right-wing politician chose a positive p, then the median voter would be jealous of her, as his utility would be lower compared to the utility of the right-wing politician. At the same time, the utility of the left-wing politician would be lower than the voter's own utility, which might even cause the median voter to feel pity for the left-wing politician. Hence the right-wing candidate would not be re-elected.

By choosing $p = 0$ instead of $-P$ or $+P$, respectively, the incumbent can decrease her own utility as well as increase the utility of the median voter and the utility of the other candidate. This equals out the difference between the median voter's utility and the utility of the incumbent and the difference between the median voter's utility and the other candidate's utility. In other words, by choosing $p = 0$ the politician ensures that the median voter is no more jealous of her than he would be of the alternative candidate if he had chosen her. If the incumbent chose a position closer to her own bliss point, then the median voter would be more jealous of the incumbent than of the other candidate and thus would not re-elect the incumbent.

6 Robustness

It is important to note that our findings hold on a very general scale. Consider the case where the function capturing inequity aversion $f(x)$ is arbitrarily small (but satisfies our assumptions). This implies that voters care very little about fairness. Even in this case, our results remain valid. Nor is it crucial for all voters to share the same notion of inequity aversion. All voters could be characterized by different functions $f_i(x)$, where some voters might like favorable inequity, i.e., $f_i(x) > 0\ \forall x > 0$ and some might dislike being better off, i.e., $f_i(x) < 0\ \forall x > 0$. This would not affect our results.[9] Our results would remain valid even if the median voter were the only one to care about fairness and all other voters did not care about inequity at all. The distribution of voters' bliss points is also irrelevant, as long as the median voter's bliss point is the average of the bliss points of both politicians. It is not crucial for the politician who runs against the incumbent at the beginning of the second period to be the same politician who lost the election in the first period. Hence, if a politician who has lost an election is replaced by a politician with identical preferences, our findings continue to hold. We could also extend our model to a multi-period setting. If we abstract from the difficulties detailed in Sect. 4 and set the discount factor δ and the

[9] In particular, one might surmise that left-wing voters are more inequity-averse than right-wing voters. This would have no impact on our findings.

"memory factor" ρ to 1 in a multi-period version of our model, then politicians would choose the median position in every period, except for the last period.[10]

7 Conclusions

Our paper gives us three insights. Firstly, fairness can have important consequences in elections as it may deliver a mechanism explaining how past behavior of politicians may affect future voter behavior. In standard models where rational utility-maximizing voters are unable to commit to a specific voting scheme, the past behavior of the government can only affect voter behavior to the extent that it reveals hidden information about characteristics of the government (e.g. about competence or preferences). In a way, we provide a microeconomic foundation for retrospective voting which is often assumed to solve the problem that past misbehavior by politicians cannot be punished by rational utility-maximizing voters. Secondly, fairness may make it more likely that moderate positions (e.g. the position of the median voter) are adopted even if parties cannot commit to future policies. Thirdly, we have shown that the very existence of inequity aversion gives the incumbent a substantial advantage in elections.

Our model also enables us to discuss the desirability of term limits. According to our model, term limits seem to be disadvantageous, since politicians choose their favorite policy in their last term in office. Compared to $\tau = 0$, which is the policy implemented by a politician angling for re-election, this is detrimental for a majority of voters.

Our paper has shown that fairness may be important for voter behavior. There are several interesting issues left for future research. In a richer model, one might discuss the optimal remuneration of politicians and how wages impact on the effort and the policy chosen by the incumbent. It would also be interesting to examine how incentive contracts for politicians, which have been introduced by Gersbach (2004), should be designed if voters are inequity averse. Considerations of fairness may reduce the need for incentive contracts if the consequences of political projects are observable. However, in the face of projects with short-term costs for voters and benefits that are observable only in the long run, voters' considerations of fairness may reduce the incentives for politicians to adopt these projects. In this case incentive contracts may be particularly beneficial.

The incumbency advantage seems to lower competition between politicians. This might be detrimental. It might also deter the entry of new, possibly very competent candidates. Thus it is interesting to establish which mechanisms might destroy the incumbent's advantage and whether it would be advantageous to adopt them.

[10] In a multi-period model, interesting institutional features such as two-period term limits could be examined.

Appendix A: Proof of Proposition 2

First we show that

$$\Delta U(p, \tau) < 0$$

if $\tau \leq 0$ and $p > 0$. This condition guarantees that a right-wing incumbent yields a higher utility for all voters with $\tau \leq 0$ if $p > 0$. Let us first introduce the following abbreviations:

$$\begin{aligned} a_1 &= 2(\tau p - \tau^2 + \tau P - Pp) \\ a_2 &= 2(\tau p - \tau^2 + \tau P + Pp + 2P^2) \\ a_3 &= 2(\tau p - \tau^2 - \tau P - Pp + 2P^2) \\ a_4 &= 2(\tau p - \tau^2 - \tau P + Pp) \end{aligned}$$

Thus we can rewrite the above inequality as

$$4P\tau + f(a_1) + f(a_2) - f(a_3) - f(a_4) < 0 \tag{8}$$

First, we analyze the case where people dislike others being worse off, i.e., $f(x) < 0 \; \forall x > 0$. We distinguish between two cases.

1. Let us for the moment assume that $-P \leq \tau \leq 0$. We note that $a_1 < 0, a_2 \geq 0$ and $a_4 \geq 0$. It is also straightforward to check that $a_4 \leq -a_1$. Thus we obtain $f(a_1) - f(a_4) < f(-a_1) - f(a_4) \leq 0$ where we have used that $f(-x) < f(x)$ $\forall x > 0$ and that $f(x)$ is strictly monotonously decreasing for $x > 0$. In addition to $-P \leq \tau \leq 0$, we now consider the case where $\tau \leq -p$. It is easy to see that $a_3 \geq a_2 \geq 0$. As $f'(x) \geq -\frac{1}{2} \; \forall x > 0$, we obtain $f(a_3) \geq f(a_2) - \frac{1}{2}(a_3 - a_2)$. This is equivalent to: $f(a_2) - f(a_3) \leq \frac{1}{2}(a_3 - a_2) = -2\tau P - 2Pp$. Since $-2\tau P - 2Pp < -4P\tau$, we have $f(a_2) - f(a_3) < -4P\tau$. Combining this inequality with $f(a_1) - f(a_4) < 0$ and $\tau \leq 0$ yields (8). Now we assume, in addition to $-P \leq \tau \leq 0$, that $\tau > -p$. If $a_3 \geq 0$, then we have $0 \leq a_3 < a_2$. It immediately follows that $f(a_2) - f(a_3) < 0$, because $f(x)$ is strictly monotonously decreasing for $x > 0$. Together with $f(a_1) - f(a_4) < 0$ we obtain (8). If $a_3 < 0$, then $a_1 \leq a_3 < 0$ and $a_2 \geq a_4 \geq 0$. Hence, we obtain $f(a_2) \leq f(a_4)$ and $f(a_1) \leq f(a_3)$. By combining these inequalities, we obtain (8) for $\tau < 0$. It is straightforward to show that (8) also holds for $\tau = 0$.
2. Let us now assume that $-1 \leq \tau < -P$. We note that $a_1 < a_3 < -a_1$. Consequently we obtain $f(a_1) < f(a_3)$. It is easy to see that $a_2 < a_4 < -a_2$. This implies $f(a_2) < f(a_4)$. By combining $f(a_2) < f(a_4)$, $f(a_1) < f(a_3)$ and $\tau \leq 0$ we can again conclude that (8) holds.

Second, we check the case with $f(x) > 0 \; \forall x > 0$, which is the case where people like being better off than other people. Note that now $f(x)$ is a strictly monotonously increasing function. Let us first consider the case $\tau \leq -p$. Since $a_3 \geq a_2$, we have $f(a_2) - f(a_3) \leq 0$. Because $a_1 < a_4$, we obtain $f(a_1) - f(a_4) < 0$.

Hence (8) again holds. Now we examine the case $\tau \leq -P$. Because $a_2 \leq a_4$ and $a_1 < a_3$, we obtain $f(a_2) \leq f(a_4)$ and $f(a_1) < f(a_3)$ which yields (8). Finally, let us assume that $\tau > -p$ and $\tau > -P$. Then it is easy to show that $a_1 < a_3 < a_2$ and $a_1 < a_4 < a_2$. We now define $f^*(x) := f(a_1) + \frac{f(a_2)-f(a_1)}{a_2-a_1}(x - a_1)$. Note that $f(a_1) = f^*(a_1)$ and $f(a_2) = f^*(a_2)$. Since $f(x)$ is a concave function, we also obtain $f^*(x) < f(x)\ \forall x \in]a_1; a_2[$. We can conclude that

$$\begin{aligned}
&f(a_1) + f(a_2) - f(a_3) - f(a_4)\\
&\quad < f(a_1) + f(a_2) - f^*(a_3) - f^*(a_4)\\
&= f(a_1) + f(a_2)\\
&\quad - \left(f(a_1) + \frac{f(a_2) - f(a_1)}{a_2 - a_1}(a_3 - a_1)\right)\\
&\quad - \left(f(a_1) + \frac{f(a_2) - f(a_1)}{a_2 - a_1}(a_4 - a_1)\right)\\
&= \left[f(a_2) - f(a_1)\right]\left(\frac{a_1 + a_2 - a_3 - a_4}{a_2 - a_1}\right) < 0
\end{aligned}$$

where we have used the fact that $a_1 + a_2 - a_3 - a_4 < 0$, $a_2 - a_1 > 0$ and $f(a_2) - f(a_1) > 0$. Hence, (8) holds. □

Appendix B: Proof of Proposition 3

One part of the proposition is the claim that the median voter with $\tau = 0$ is indifferent between both candidates if $p = 0$. This can be easily verified by observing that $\Delta U(0, 0) = 0$ follows from $\Delta U(p, \tau) = -\Delta U(-p, -\tau)$.

Now we prove that all voters with $\tau > 0$ strictly prefer voting for the right-wing candidate at the beginning of the second period if the incumbent has chosen $p = 0$ in the first period.

We distinguish between two cases. First, we consider the case where advantageous inequity is desirable from the perspective of voters, i.e., $f(x) > 0\ \forall x > 0$. Using the definitions introduced in the proof of Proposition 2, we state that $a_1 > a_4$ and $a_2 > a_3$ for $\tau > 0$ and $p = 0$. This implies $f(a_2) - f(a_3) > 0$ and $f(a_1) - f(a_4) > 0$. Thus we obtain $4P\tau + f(a_1) + f(a_2) - f(a_3) - f(a_4) > 0$.

Second, we consider the case where advantageous inequity reduces the overall utility of voters, i.e., $f(x) < 0\ \forall x > 0$.

1. Assume $0 < \tau \leq P$. Note that $a_4 < 0$, $a_1 \geq 0$ and $-a_4 > a_1$. This implies $f(a_1) - f(a_4) > f(a_1) - f(-a_4) > 0$.
   Note that $a_2 > 0$, $a_3 \geq 0$ and $a_2 > a_3$. Since $f'(x) \geq -\frac{1}{2}$, we can conclude $f(a_2) \geq f(a_3) - \frac{1}{2}(a_2 - a_3) = f(a_3) - \tau P > f(a_3) - 4\tau P$.
   Combining these inequalities yields: $4P\tau + f(a_1) + f(a_2) - f(a_3) - f(a_4) > 0$.
2. Assume $P < \tau \leq 1$. Then $a_1 < 0$, $a_3 < 0$ and $a_3 < a_1$. Thus, the following inequality holds: $f(a_1) - f(a_3) > 0$.

We also observe that $a_4 < 0$. If $P < \tau \le 2P$, then $a_2 \ge 0$ and $-a_4 > a_2$, which implies $f(a_2) - f(a_4) > f(a_2) - f(-a_4) > 0$. If $\tau > 2P$, then $a_2 < 0$ and $a_2 > a_4$, which also implies $f(a_2) - f(a_4) > 0$.
Thus we obtain $4P\tau + f(a_1) + f(a_2) - f(a_3) - f(a_4) > 0$.

Hence, all voters with $\tau > 0$ always vote for the right-wing candidate in the second period if she has chosen policy $p = 0$ in the first period. □

Appendix C: Proof of Proposition 4

It suffices to show that $\partial \Delta U / \partial p$ is negative for $p = 0$ and $\tau = 0$. This implies that the median voter is strictly better off when voting for a right-wing incumbent if the right-wing incumbent chooses a slightly negative p. Since ΔU is strictly positive for $\tau > 0$ and $p = 0$, it follows that all voters with $\tau > 0$ also strictly prefer voting for a right-wing incumbent when p is slightly negative.

For $p = 0$ and $\tau = 0$ we obtain

$$\frac{\partial \Delta U}{\partial p} = -2Pf'(0) + 2Pf'(4P^2) - f'(4P^2)(-2P) - f'(0)(2P) \tag{9}$$

$$= 4P\left(f'(4P^2) - f'(0)\right) \tag{10}$$

This expression is negative since f is a concave function. □

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